

Cosmic D -strings as Axionic D -term Strings

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Abstract

In this work we derive non-singular BPS string solutions from an action that captures the essential features of a D -brane-anti- D -brane system compactified to four dimensions. The model we consider is a supersymmetric abelian Higgs model with a D -term potential coupled to an axion-dilaton multiplet. The strings in question are axionic D -term strings which we identify with the D -strings of type II string theory. In this picture the Higgs field represents the open string tachyon of the $D - \bar{D}$ pair and the axion is dual to a Ramond Ramond form. The crucial term allowing the existence of non-singular BPS strings is the Fayet-Iliopoulos term, which is related to the tensions of the D -string and of the parent branes. Despite the presence of the axion, the strings are BPS and carry finite energy, due to the fact that the space gets very slowly decompactified away from the core, screening the long range axion field (or equivalently the theory approaches an infinitely weak $4D$ coupling). Within our $4D$ effective action we also identify another class of BPS string solutions (s -strings) which have no ten dimensional analog, and can only exist after compactification.

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1 Overview

It was suggested long ago by Witten [1] that fundamental superstrings (F-strings) of macroscopic length could be observed in the form of cosmic strings. After the discovery of D -branes [2], it is natural to expect that a similar role could be played by other extended objects of string theory such as D_{1+q} -branes wrapped around internal q -cycles. It is interesting that brane inflation [3] generically predicts the formation of such objects, whereas the formation of point-like or wall-like extended objects is suppressed [4]. Various aspects of the dynamics, formation and evolution of cosmic F - and D -strings have been discussed in [5–10]. Needless to say that a possible observation of these objects would provide a direct window into string theory.

Thus, both from an observational as well as from the fundamental point of view it is important to understand the precise nature and structure of the *stringy* cosmic strings. This is the motivation that led to the present work. We shall derive non-singular BPS string solutions which resemble many of the features of the D -strings. Following the conjecture in [5], which we review below, we interpret our solutions as the D -strings of string theory. Independently from the conjecture the solutions presented in this paper are new BPS objects which have interest of their own: they are the first example of finite energy cosmic strings coupled to an axion field.

In trying to understand the $4D$ picture of the D -strings, it is useful to consider their description in terms of Sen’s tachyonic vortices formed on the worldvolume of a higher dimensional unstable D -brane-anti- D -brane ($D - \bar{D}$) pair [11]. For example, a D_1 -brane in ten dimensions can be viewed as a tachyonic vortex on the worldvolume of a $D_3 - \bar{D}_3$ pair. This vortex originates as follows. The gauge symmetry of the system consists of two $U(1)$ ’s belonging to the worldvolume theories of the D_3 and \bar{D}_3 respectively. The tachyon (ϕ), which is an excitation of an open string stretched between the two branes, is charged under the diagonal combination of the two $U(1)$ ’s. When the branes annihilate, the tachyon condenses and this gauge symmetry is Higgsed. Since the tachyonic vacuum is topologically non-trivial [12], there are topologically stable vortex configurations, analogous to Abrikosov-Nielsen-Olesen [13] flux tubes, which carry magnetic flux of the Higgsed $U(1)$. These flux tubes are the D_1 -strings. In this picture we can understand their Ramond-Ramond (RR) charge as originating from the Wess-Zumino coupling on the $D_3 - \bar{D}_3$ worldvolume¹,

$$\int_{3+1} F_2 \wedge C_2, \tag{1.1}$$

where C_2 is the RR two-form and F_2 denotes the field strength of the diagonal $U(1)$ gauge field. After compactification to four dimensions D_1 -strings become cosmic strings. Since in

¹In string theory descriptions the RR charge may also be reproduced by couplings of the tachyon to C_2 .

ten dimensions D -branes preserve half of the supersymmetries, in $4D$ it should be possible to find some kind of cosmic D -strings which are still BPS saturated objects.

It is therefore natural to ask whether there are such solitons in $4D$ supergravity. In [5] it was shown that the only BPS gauge strings in supergravity are D -term strings. These are the strings that are formed by Higgsing a $U(1)$ gauge symmetry due to the presence of a Fayet-Iliopoulos (FI) D -term. Because of this fact, it was conjectured that D -strings, if they have any solitonic counterpart in the effective four dimensional field theory description, must be represented by some form of BPS D -term strings.

The " *D -string D -term-string equivalence*" conjecture leads to the conclusion that the energy density stored in an unstable $D - \bar{D}$ pair is a D -term associated with the FI term of the worldvolume $U(1)$ that is Higgsed by the tachyon. Therefore, in the ten dimensional limit, the effective potential for the tachyon is schematically,

$$V_D = \frac{g^2}{2}(\xi - |\phi|^2 + \dots)^2 \quad (1.2)$$

where the ellipses stand for all the other charged fields in the system. These fields have positive mass squared and vanish throughout the annihilation process. It follows from (1.2) that the FI term ξ is related to the D_3 brane tension by

$$\frac{g^2 \xi^2}{4} = T_3 \quad (1.3)$$

where T_3 is the D_3 tension and g is the $U(1)$ gauge coupling constant. In the ten dimensional limit, the annihilation proceeds solely through the tachyon condensation, which compensates the FI term in (1.2). The resulting D -strings are purely tachyonic vortices, carrying ten dimensional RR charge. Under the term *purely tachyonic* what is meant here is that the only winding phase responsible for the topological stability of the string, and consequently for the existence of the magnetic flux, is the phase of the tachyon. From eq. (1.1) it follows that the electric RR charge of the D_1 -brane arises from the quantized magnetic flux carried by the vortex.

The effect of compactification on the above system is rather profound. First, after dimensional reduction we are left with the zero mode of the RR two-form, C_2 , which in four dimensions is dual to an axion field a . In a supersymmetric scenario the axion has a scalar partner s .² Dualizing the two-form one generically obtains the following term in the action

$$\frac{M_P^2}{s^2} (\partial_\mu a + 2 \delta A_\mu)^2, \quad (1.4)$$

²Depending on the details of the compactification s is related to the dilaton and the volume modulus. In general the axion of the four dimensional theory arises from the zero mode of a Ramond-Ramond field $C_{\mu\nu i \dots j}$ with two of the indices in the large dimensions.

where $\delta \approx \xi/M_P^2$ and M_P is the four dimensional Planck mass. From this we see that the Wess-Zumino term (1.1) gauges the shift symmetry of the axion. From eq. (1.4) it is also clear that after compactification, the worldvolume $U(1)$ symmetry is always in the Higgs phase, even when the tachyon vacuum expectation value (VEV) is set to zero. The mass of the gauge field of course vanishes in the infinite space limit, as $\delta \rightarrow 0$. Another effect of the compactification is that the D -term potential (1.2) acquires an s -dependent contribution,

$$V_D = \frac{g^2}{2} \left(\xi - |\phi|^2 - \frac{\delta M_P^2}{s} \right)^2. \quad (1.5)$$

Notice that this correction too dies off in the decompactified limit. This fact is consistent with the picture that in ten dimensions D -strings are purely tachyonic vortices whose tension is set by the FI term ξ .

Previous attempts to derive solutions for D -strings as BPS solitons in four-dimensions were in the following directions. As we have already mentioned, in [5] smooth BPS D -term string solutions were derived in the presence of a FI term ξ , but the dilaton-axion multiplet was not included in the solution. Notice however that this is consistent with supersymmetrically removing this multiplet as shown in [8]. In [16, 17] axionic string solutions were studied in a model in which the $U(1)$ anomalies are cancelled by Green-Schwarz (GS) mechanism. Although this model is primarily motivated by the heterotic $E_8 \times E_8$ compactification, it nevertheless shares some similarities with the type II string compactifications that are our main interest. The low energy effective actions are very similar, with the crucial difference that in the heterotic theory the existence of a dilaton-independent part of FI term ξ is not obvious and was not included in [16, 17]. As a result, the string solutions found in these models are either singular or break all the supersymmetries.

In this paper we show that the presence of the ξ term in (1.5) allows us to construct smooth BPS D -string solutions. Other than being new interesting BPS objects, their existence provides an additional support for the conjecture of [5]. Some complementary string theoretic evidence for this conjecture was also provided in [15], where D -term strings were identified in intersecting brane systems. A discussion of D -term strings in the context of wrapped branes has also appeared in [14]. Let us also mention that, while we only study in detail supersymmetric strings, the BPS properties of D -strings on a background such as flux compactifications [7] or the deformed conifold [18] may be lost.

In the present work we focus on explicit field theoretic solutions for D -strings and then interpret them in the framework of D -brane systems. Actually we find two different types of smooth BPS cosmic string solutions, both of which have an interpretation in string theory:

ϕ -strings (D -strings)

The first type of solutions, which fits all the features of the D -strings, is the tachyonic vortex. In this case the magnetic flux is induced by the winding of the tachyon phase, and not by the axion. At large distances from the core the asymptotic values of the fields for this solution are,

$$\phi \rightarrow \xi e^{in\theta}, \quad s \rightarrow \infty, \quad (1.6)$$

where θ is the angular coordinate in the plane perpendicular to the string. As we will show the asymptotically infinite value of s is the only way to avoid the logarithmic divergence of the energy in this case. Such a divergence would be incompatible with the BPS condition. Since s represents a combination of the dilaton and the volume modulus, the space gets decompactified (or the $4D$ effective coupling goes to zero) at large distances from the core. This effect however happens very slowly since s depends only logarithmically on the distance.

s -strings

In the second class of solutions the tachyon VEV vanishes at infinity, and the magnetic flux is induced by the winding of the axion. s now goes asymptotically to a constant in such a way to compensate the ξ term in the potential (1.5). These strings can be interpreted as the vortices produced by Higgsing the $U(1)$ group of a $D - \bar{D}$ pair by compactification, as opposed to tachyon condensation. Thus, far away from the string core the tachyon mass is zero and the branes do not annihilate. The tachyon may only develop an expectation value in the core of the string. The existence of a BPS configuration on a background with a $D_3 - \bar{D}_3$ pair may come as a surprise, because in ten-dimensions this background breaks all the supersymmetries (for an exception see [19]). However, the identification of the $D_3 - \bar{D}_3$ tension with the D -term energy density suggests that tachyon condensation may not be the only way for restoring part of the supersymmetries once the effects of the compactification are taken into account. Instead, the FI term may be compensated by the dilaton s . The corresponding BPS strings are then the s -strings.

This paper is organized as follows. In section 2 we describe the supersymmetric models that we will be considering. In section 3 we derive the Bogomol'nyi equations for the string configurations. In section 4 we give a detailed description of the two types of cosmic string solutions in global as well as local supersymmetry. In section 5 some new features of the string solutions are discussed. In section 6 we interpret our solutions within the context of string theory. We end with some conclusions and future directions.

2 The model

For the globally supersymmetric case the effective $N = 1$ lagrangian that we will consider takes the following form in superspace (we follow the conventions in [20]),

$$L = \int d^4\theta \left[\Phi_i^\dagger e^{-q_i V} \Phi_i + K(S + \bar{S} + 4\delta V) + 2\xi V \right] + \int d^2\theta \frac{1}{4} f(S) W^\alpha W_\alpha + h.c. \quad (2.1)$$

Φ_i are chiral fields with integer $U(1)$ charges q_i . Since the $U(1)$ symmetry is linearly realized on the Φ_i for simplicity we will consider the minimal Kähler potential for this superfield. The axion is contained in a chiral superfield S whose lowest component is $s + ia$ where s is (loosely speaking) the dilaton. Invariance of the action under a constant shift of the axion demands that the Kähler potential K is a function of $S + \bar{S}$. This shift symmetry is gauged under the $U(1)$. In the string motivated models that we have in mind one typically finds Kähler potentials of the form $K = -M_P^2 \log [S + \bar{S}]$ (up to a numerical coefficient) so we will study this case in detail even though most of our results hold for any $K(S + \bar{S})$.

As far as the string theory motivation is concerned, we will use (2.1) as the prototype lagrangian describing interactions of the open string tachyon ϕ (represented by lowest component of one of the Φ_i) and the complexified RR axion $s + ia$ in type II compactifications with $D - \bar{D}$ background. We shall show that these fields are sufficient to describe non-singular BPS D -string solutions. It is important to stress that (2.1) is not the complete low energy action, because a $D - \bar{D}$ system contains other light fields, e.g., such as the brane-separation mode. However, these additional fields can be consistently ignored.

Note that the action above is similar to the one of $E_8 \times E_8$ heterotic string compactifications in which the $U(1)$ anomalies are cancelled by the Green-Schwarz mechanism, but with the important difference that we include a constant (moduli independent) FI term ξ . For type II compactifications, such constant term is necessary in order to account for the $D - \bar{D}$ tension, which should survive in the ten-dimensional limit. As we shall show this term is crucial for the existence of non-singular BPS D -string solutions. In this respect our action differs from the one considered in [16, 17] in which no ξ term was included, and consequently no smooth BPS string solutions were found. We do not include a superpotential, since BPS requirement demands that F -terms must be identically zero on the string solution [5].

In components the bosonic part of the action is,

$$\begin{aligned} L &= -|D_\mu \phi_i|^2 - K_{S\bar{S}} |D_\mu S|^2 - \frac{1}{4} \text{Re}(f) F^{\mu\nu} F_{\mu\nu} + \frac{1}{4} \text{Im}(f) F^{\mu\nu} \tilde{F}_{\mu\nu} - \frac{1}{2} \text{Re}(f) D^2 \\ D &= -\frac{1}{\text{Re}(f)} \left(2\delta K_S - \sum_i q_i |\phi_i|^2 + \xi \right) \end{aligned} \quad (2.2)$$

with the covariant derivatives given by,

$$\begin{aligned} D_\mu \phi_i &= (\partial_\mu - iq_i A_\mu) \phi_i \\ D_\mu S &= \partial_\mu S + 2i\delta A_\mu. \end{aligned} \tag{2.3}$$

Under a gauge transformation the bosonic fields transform as,

$$\begin{aligned} \phi_i &\rightarrow e^{iq_i \lambda} \phi_i \\ A_\mu &\rightarrow A_\mu + \partial_\mu \lambda \\ a &\rightarrow a - 2\delta \lambda. \end{aligned} \tag{2.4}$$

From the last equation we deduce that the periodicity of the axion is,

$$[a] = 4\pi\delta. \tag{2.5}$$

Depending on the charges q_i we can distinguish two cases. When the $\sum_i q_i = 0$ the matter sector of the lagrangian is anomaly free. Gauge invariance of the action then fixes $f(S)$ to be a constant and δ to be arbitrary. When the sum of the charges is different from zero the theory has mixed anomalies. In any consistent theory all the gauge anomalies must vanish. In this case the anomaly can be cancelled by the Green-Schwarz mechanism applied to four dimensions [21] by choosing the gauge kinetic function $f(S) = S$. The axion has a universal coupling of the form $a F \tilde{F}$. Under the gauge transformation the axion shifts and generates a contribution to the anomaly which cancels the one of the matter sector. Clearly this implies severe restrictions on the representations which are automatically satisfied in string compactifications. In heterotic string compactification one finds for example³,

$$\delta = \frac{1}{192\pi^2} \sum_{i=1}^n q_i. \tag{2.6}$$

For the main part of this paper we will focus on the scenario with constant gauge kinetic function, $f(S) = 1/g^2$ and $K = -M_P^2 \log[S + \bar{S}]$ which is relevant for type II compactifications. The pseudo-anomalous case requires only minor modifications. The vacuum manifold corresponds to the zeros of the D -term in (2.2),

$$M_P^2 \frac{\delta}{s} + \sum_{i=1}^n q_i |\phi_i|^2 = \xi \tag{2.7}$$

³We should note here that we are not aware of a mechanism to generate a s independent FI term in heterotic string theory. On the other hand in type II theories the FI term should account for the energy density of the $D - \bar{D}$ pair.

Due to the fact that the axion shifts under the $U(1)$ gauge transformation the theory is always in the Higgs phase in any vacuum. In general the gauge boson eats a linear combination of the axion and of the phase of the Higgs field while the orthogonal combinations remain massless. The mass of the vector boson is given by,

$$m^2 = g^2 \left(\frac{4\delta^2}{s^2} + \sum_{i=1}^n q_i^2 |\phi_i|^2 \right) \quad (2.8)$$

with s and ϕ_i lying in the vacuum manifold (2.7). In the supersymmetric vacuum the gauge boson belongs to a massive $N = 1$ multiplet.

3 Bogomol'nyi equations

We consider solitonic string solutions with the fields only varying in the transverse plane (x, y) . Remarkably for *any* Kähler potential of the form $K(S + \bar{S})$ the energy density of this configuration can be organized in the Bogomol'nyi form,

$$\begin{aligned} E &= \int d^2x |(D_x \pm iD_y)\phi_i|^2 + K_{S\bar{S}} |(D_x \pm iD_y)S|^2 + \frac{1}{2} \text{Re}(f)(F_{xy} \pm D)^2 \\ &\pm \xi \int d^2x F_{xy}. \end{aligned} \quad (3.1)$$

where we have integrated by parts assuming that the boundary term at infinity is zero and is understood that the signs are chosen so that the energy is positive. From this we can directly read off the Bogomol'nyi equations,

$$(D_x \pm iD_y)\phi_i = 0 \quad (D_x \pm iD_y)S = 0 \quad F_{xy} \pm D = 0 \quad (3.2)$$

where the covariant derivatives are given in (2.3). When the Bogomol'nyi equations are satisfied the tensions of these solutions is minimal and proportional to the magnetic flux through the string,

$$T = \pm \xi \int d^2x F_{xy}. \quad (3.3)$$

The minimization of the energy guarantees that the equations of motion are also satisfied.

Generically in a supersymmetric theory the Bogomol'nyi energy bound signals that the solution leaves a fraction of the supersymmetries unbroken, i.e. it is a BPS state. The same holds here. A configuration of the fields preserves some supersymmetries if the variations of all the fermions is zero for some projection of the supersymmetry parameters. The transformations

of the fermions under supersymmetry evaluated on the bosonic background are given by,

$$\begin{aligned}\delta_\epsilon \psi_i &= i\sqrt{2}\sigma^\mu \bar{\epsilon} D_\mu \phi_i \\ \delta_\epsilon \chi &= i\sqrt{2}\sigma^\mu \bar{\epsilon} D_\mu S \\ \delta_\epsilon \lambda &= \sigma^{\mu\nu} \epsilon F_{\mu\nu} + i\epsilon D\end{aligned}\tag{3.4}$$

where ψ_i and χ are the fermionic partners of the scalar fields and λ is the gaugino. It is straightforward to check that, when the Bogomol'nyi equations (3.2) are satisfied, the solutions leave half of the supersymmetries unbroken, so as expected these are BPS states. This guarantees their stability even quantum mechanically. Supersymmetry immediately explains why the Bogomol'nyi equations do not depend on the Kähler potential (except through the D -term), giving a rationale for why it was possible to rearrange the energy in the simple Bogomol'nyi form (3.1). It is also easy to see considering the supersymmetry variations, that in the presence of a non zero F -term all the supersymmetries must be broken. Therefore only D -term strings can be BPS objects [5]. The only exception to this statement is when the superpotential generating the F -term is chosen such that the system has $N = 2$ supersymmetry because in this case there is no distinction between F and D terms [22]. However this mechanism does not seem viable in supergravity because it appears to be impossible to construct supersymmetric actions with constant FI terms in $N = 2$ supergravity with charged hypermultiplets [23].

It is important to stress that when the BPS equations are satisfied the tension is equal to the constant FI term ξ times the magnetic flux. This shows that any attempt to find smooth BPS configurations with a field dependent FI term is bound to fail because this configuration would have zero energy and therefore it can only be the vacuum. Of course this leaves the possibility to build singular BPS string solutions.

4 String solutions

In this section we present the BPS string solutions of our model for the relevant case $K = -M_P^2 \log[S + \bar{S}]$. For simplicity we first consider the globally supersymmetric case (here we set $M_P = 1$). We find two classes of string solutions, the ones created by the Higgs field ϕ and the ones created by the dilaton s which have rather different properties. We consider in some detail the case with constant gauge kinetic function (corresponding to no anomalies). The case where anomalies are cancelled by the GS mechanism is slightly more complicated but leads to qualitatively similar results that are briefly reported in section 4.3. Finally we show that our solutions can also be coupled to supergravity preserving their BPS nature.

4.1 The ϕ -strings (D -strings)

These are the solutions where one of the fields ϕ_i goes to a finite value at infinity. In order to solve the BPS equations for a string configuration we take the following ansatz for the fields⁴,

$$\begin{aligned}\phi_1 &= f(r)e^{in\theta} \\ S &= s(r) - 2i\delta m\theta \\ A_\theta &= n\frac{v(r)}{r}.\end{aligned}\tag{4.1}$$

We assume for simplicity that all the scalar fields except ϕ_1 are equal to zero. One could construct more general BPS strings where several ϕ_i 's are different from zero. In fact our solutions are reminiscent of semilocal strings [28] and the relation will be clarified in section 5. Notice that, while it is consistent with the BPS equations (see below) to set the other chiral fields to zero, s cannot be a constant in a supersymmetric configuration because in the BPS equations the dilaton is sourced by the gauge field.

Using the ansatz above for the fields the BPS equations read,

$$\begin{aligned}f' &= |n|\frac{1-qv}{r}f \\ s' &= -2\delta\frac{|m|-|n|v}{r} \\ |n|\frac{v'}{r} &= g^2\left(\xi - \frac{\delta}{s} - qf^2\right),\end{aligned}\tag{4.2}$$

where we have assumed δ and ξ to be positive and we have set $f(S) = 1/g^2$. In general the first two equations imply the following relation between s and f ,

$$s = -2\delta\left(|m| - \frac{|n|}{q}\right)\log r - \frac{2\delta}{q}\log f + k\tag{4.3}$$

where k is an arbitrary integration constant related to the size of the core of the defect. Taking into account the fact that close to the origin $f \sim \alpha r^{|n|}$ (which follows from the first equation in (4.2)) and that f goes to a constant at infinity one derives,

$$\begin{aligned}s &\sim -2\delta|m|\log r & \text{as } r \rightarrow 0 \\ s &\sim -2\delta\left(|m| - \frac{|n|}{q}\right)\log r & \text{as } r \rightarrow \infty\end{aligned}\tag{4.4}$$

At infinity s always goes to infinity unless the winding of the axion is q times the one of the phase of the tachyon. In this case, there is family of solutions labeled by the asymptotic value of f . In all other cases the boundary conditions at infinity are $f(\infty) = \sqrt{\xi/q}$ and $v(\infty) = 1/q$.

⁴The ansatz for the axion is chosen to respect the periodicity obtained in (2.5).

Restrictions on the possible values of n and m can be derived from eqs.(4.4). In any configuration with finite energy the dilation s should never cross zero. Also, $s \rightarrow 0$ corresponds to strong coupling in our effective theory so we would not trust our solutions in that regime. Requiring that s is always positive leads to,

$$q|m| \leq |n| \quad (4.5)$$

which fixes the charge q to be positive. If q is much bigger than one the only BPS solution is with the axion not winding, $m = 0$. When δ is negative (the sign is determined a priori when the $U(1)$ symmetry is anomalous) solutions can of course still be found but in this case we need to choose a field with negative electric charge and ξ to be negative. In a given theory the string solutions can only exist if δ and ξ have the same sign so in principle the coupling to the axion multiplet might destroy the BPS solutions. In figure 4.1 we plot the string profiles for $n = 1, m = 0$ and $n = 2, m = 1$.

The asymptotic expansion for the Higgs and the gauge fields can be derived by solving the linearized equations of motion at infinity. To leading order one finds,

$$\begin{aligned} f &= \sqrt{\frac{\xi}{q}} - \frac{\sqrt{q}}{4\sqrt{\xi}(|n| - q|m|)} \frac{1}{\log r} + \dots \\ v &= \frac{1}{q} - \frac{1}{4|n|\xi} \frac{1}{(|n| - q|m|)} \frac{1}{(\log r)^2} + \dots \end{aligned} \quad (4.6)$$

These expansions should be compared with the ordinary abelian Higgs model where the fields approach their asymptotic values exponentially fast. In comparison to the abelian Higgs model, where the energy is localized on scales comparable to the inverse Higgs mass, here the energy is spread on larger scales (depending on k). Asymptotically our solutions resemble axionic strings but the BPS equations guarantee that the energy remains finite. The tension of the string is given by,

$$T = \pm \xi \int d^2x F_{xy} = \frac{2\pi|n|\xi}{q} \quad (4.7)$$

As this formula shows the tension depends only on the winding of the Higgs field ϕ and not on the winding of a .

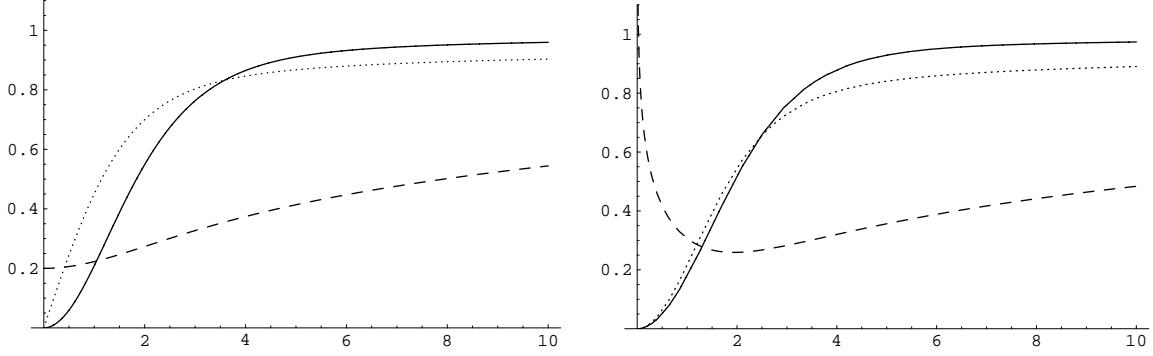


Figure 1: **Plot of the ϕ -string profiles.** The figure on the left corresponds to the case $n = 1$, $m = 0$ described in the main text, while the one on the right corresponds to $n = 2$, $m = 1$. We have taken the following set of parameters to obtain these solutions numerically, $g = q = \xi = 1$ and $\delta = 1/10$. The solid line denotes the function $v(r)$ that parametrizes the magnetic field profile, the dashed line is the dilaton field $s(r)$ and the dotted line is the Higgs field $\phi(r)$.

4.2 The "s-strings"

Very different solutions can be obtained when the string is supported by the field s going to a constant. In this case the ansatz for the fields is,

$$\begin{aligned}\phi_1 &= f(r)e^{in\theta} \\ S &= s(r) - 2i\delta m\theta \\ A_\theta &= m\frac{v(r)}{r}.\end{aligned}\tag{4.8}$$

This is almost identical to the previous case with the only difference that the winding of the gauge field is now linked to the one of the axion field. The BPS equations become,

$$\begin{aligned}f' &= \frac{|n| - q|m|v}{r}f \\ s' &= -2\delta|m|\frac{1-v}{r} \\ |m|\frac{v'}{r} &= g^2\left(\xi - \frac{\delta}{s} - qf^2\right)\end{aligned}\tag{4.9}$$

From the first two equations it follows that, as in the previous case, s and f are related by $s = -2\delta(|m| - |n|/q)\log r - 2\delta/q\log f + k$. This equation now requires that f goes to zero at infinity since s goes to a finite value. The other boundary conditions which can be deduced from the BPS equations are $v(\infty) = 1$ and $s = \delta/\xi$. Numerical solutions are shown in Fig.(2).

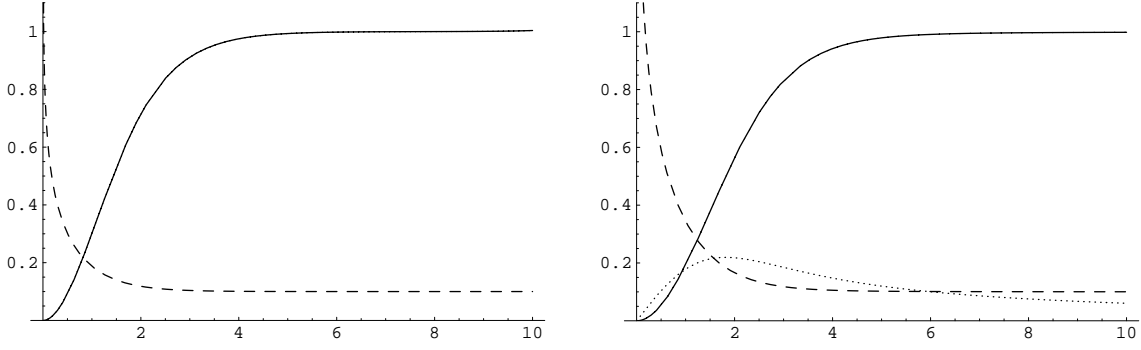


Figure 2: **Plot of the s -string profiles.** The figure on the left shows an s -string with $\phi = 0$ and $m = 1$. The one on the right displays the case with $m = 2$ and $n = 1$. We use the same parameters as in Fig.1.

The tension of the s -strings is given by,

$$T = \pm \xi \int d^2x F_{xy} = 2\pi|m|\xi. \quad (4.10)$$

In general the tension of the s -strings is q -times the one of the ϕ strings. The tension now depends only on the winding of the axion and we have degenerate solutions with different values of n . Let us consider the asymptotic expansion of these solutions. If $f = 0$ the linearized equations for s and v are identical to the ones for the abelian Higgs model so the fields approach the asymptotic values exponentially. For $f \neq 0$ differently from the ϕ -strings the fields approach the asymptotic values power like,

$$\begin{aligned} f &= \frac{a}{r^{q|m|-|n|}} + \dots \\ s &= \frac{\delta}{\xi} + \frac{\delta}{\xi^2} \frac{a^2}{r^{2q|m|-2|n|}} + \dots \\ v &= 1 + \frac{|n| - q|m|}{|m|\xi^2} \frac{a^2}{r^{2q|m|-2|n|}} + \dots \end{aligned} \quad (4.11)$$

As for the ϕ -strings the relation between s and f leads to restrictions on the values of m and n , namely $q|m| > |n|$ (this simply follows from the fact that since s goes to a constant f must go to zero).

In the special case $q|m| = |n|$, the ϕ and s -strings have the same tension. In fact for these values of the parameters the two string solutions become degenerate and it is possible to construct more general solutions with the fields approaching any point of the vacuum manifold (2.7).

4.3 Pseudo-anomalous $U(1)$ strings

Finally let us turn to the pseudo-anomalous case. At least in some compactification the axion acquires an effective coupling,

$$a F_2^A \wedge F_2^A, \quad (4.12)$$

where A runs over some gauge group factors G_A . The existence of such a coupling signals that the fermion content of the theory has a mixed $U(1) \times G_A^2$ anomaly, which is cancelled by the Green-Schwarz mechanism.

The coupling (4.12) has a double effect on the D -strings solutions in question. First, it modifies the structure of the D -term potential, and secondly, because of instantons, it may lead to the attachment of the domain walls to the strings. The D -term potential is modified, because in the case of GS anomaly cancellation $f(S) = S$. Using the same ansatz for the fields as in the previous cases the first two BPS equations remain unchanged while the last equation becomes,

$$|n| \frac{v'}{r} = \frac{1}{s} \left(\xi - \frac{\delta}{s} - q f^2 \right), \quad (4.13)$$

where n is the winding of the phase of the tachyon or of the axion for the ϕ -strings and for the s -strings respectively.

Let us first briefly discuss the ϕ -strings. The vacuum manifold is the same as in (2.7) plus the point $s = \infty$ (for any value of f). s diverges at infinity for the same reasons explained in section 4.1. This would in principle still allow f to go to a constant different from one at infinity. However this is incompatible with the equation above since there are no solution where v is bounded unless $f(\infty) = 1$ (this can easily be shown using the fact that $s \sim \log r$ asymptotically). The solutions of the BPS equations are qualitatively very similar to the ones with constant gauge kinetic function. In fact to leading order the asymptotic behavior of the solutions is as in (4.6). For the anomalous s -strings the situation appears even simpler. Since s goes to a constant at infinity the solutions are almost identical the ones in section 4.2.

Let us turn to the issue of the domain walls now. The instanton effects may generate a potential for the axion and the tachyon phase. For instance, in the presence of the coupling (4.12), the instantons in question may be the gauge theory instantons in one or more of the G_A groups. The precise nature of the potential is model dependent, but the important fact is that the potential is only generated for the gauge-invariant combination of phases

$$V(a + 2\Theta\delta) \quad (4.14)$$

Thus, only the strings for which the combination $a + 2\Theta\delta$ winds at infinity can potentially become boundaries of the domain walls. For the heterotic F -strings the possibility of becoming

boundaries of domain walls was suggested by Witten [1] and for the D -strings such a possibility was discussed by Copeland, Myers and Polchinski [7]. Presumably, the same physics that generates the potential (4.14) will also lift $\phi - s$ flat direction. In such a case our ϕ - and s -string solutions will acquire a logarithmically divergent energy, and will become boundaries of the usual axion type domain walls.⁵

4.4 Coupling to supergravity

In this section we briefly show how the string solutions presented in the previous sections can be coupled to supergravity.

Localized lumps of energy with codimension two create an asymptotically conical space-time with a deficit angle at infinity proportional to the tension of the object. At first sight this suggests that all supersymmetries must be broken when gravity is dynamical because covariantly constant spinors cannot be defined on a conical space. This is the same problem faced and solved in the case of the pure abelian Higgs model vortices [5, 24] (see also [25] for related work). In the presence of a FI term the gravitino and the supersymmetric parameter ϵ become charged under the $U(1)$ gauge symmetry so that the gravitino supersymmetric variation has an extra contribution proportional to the gauge connection which cancels exactly the one due to the deficit angle. In this section we generalize the analysis of the pure abelian Higgs model to a scenario with an arbitrary Kähler potential and show that in general a BPS string configuration in global supersymmetry remains BPS when supergravity is (weakly) coupled.

We follow closely the analysis in [24] and refer the reader to this paper for the details of the derivation. We take the following ansatz for the metric,

$$ds^2 = -dt^2 + dz^2 + e^{2\rho(x,y)} (dx^2 + dy^2). \quad (4.15)$$

In most general case [20] one can consider several chiral fields A_i transforming under the $U(1)$ symmetry where the transformation corresponds to an isometry of the scalar manifold. In a bosonic background the variations of the fermions read,

$$\begin{aligned} \delta_\epsilon \chi_i &= i\sqrt{2}\sigma^\mu \bar{\epsilon} D_\mu A_i \\ \delta_\epsilon \lambda &= \sigma^{\mu\nu} \epsilon F_{\mu\nu} + i\epsilon D \\ \delta_\epsilon \psi_\mu &= \left(\partial_\mu - \frac{i\xi}{2M_P^2} A_\mu \right) \epsilon - \epsilon \omega_\mu - \frac{1}{4M_P^2} J_\mu \epsilon. \end{aligned} \quad (4.16)$$

⁵One could also imagine a situation in which the $\phi - s$ flat direction is not lifted by the axion-potential. In such a case the domain walls should "dissolve" at infinity, because the potential should vanish in either of the two limits $|\phi| \rightarrow 0$ and for $s \rightarrow \infty$. This follows from the fact that in both of these limits the physical gauge invariant axion becomes ill-defined.

where $D_\mu A_j$ are the covariant derivatives of the scalar fields in the theory which can be expressed in terms of the Killing vectors of the scalar manifold. For a general Kähler potential the current J_μ is given by,

$$J_\mu = K_j D_\mu A^j - K_{j^*} D_\mu A^{*j}. \quad (4.17)$$

It is important to observe that since the gravitino is charged under the $U(1)$ its supersymmetric variation contains a piece proportional to the gauge connection. This is what allows to define Killing spinors in an asymptotically conical space.

The supersymmetry transformations of the matter fields are as in flat space with suitable factors of the metric. The BPS equations then become,

$$(D_x \pm iD_y)A_i = 0 \quad e^{-2\rho} F_{xy} \pm D = 0. \quad (4.18)$$

Since the first equation does not depend on the metric explicitly, for the model considered in this paper the same relation between s and f found in (4.3) holds.

Supersymmetry is unbroken by the background if the Killing spinor equations $\delta\psi_\mu = 0$ admits solutions. These equations imply the integrability condition [24],

$$-M_P^2 \square \rho = \pm \xi F_{xy} + \frac{i}{2} (D_x J_y - D_y J_x) \quad (4.19)$$

where \square is the two dimensional laplacian. This equation determines the conformal factor ρ , therefore what remains to be checked is that this equation is consistent with the Einstein's equations for the metric. This is indeed the case as one can check that (4.19) is simply the 00 Einstein equation for the metric when the BPS equations (4.18) are imposed. The other equations of motion are also satisfied as the energy can still be cast in the Bogomol'nyi form (3.1) (with appropriate metric factors). This concludes the proof that the BPS string solutions studied in this paper can be lifted to supergravity.

At large distance from the core of the string eq. (4.19) can be easily integrated since (J_x, J_y) go to zero faster than $1/r$ (this assumption is necessary to write the energy in the Bogomol'nyi form and is satisfied in our model). One finds that $e^{2\rho} = (x^2 + y^2)^{-T/(2\pi M_P^2)}$. After a simple change of variables to polar coordinates the metric becomes,

$$ds_{r \rightarrow \infty}^2 = -dt^2 + dz^2 + dr^2 + r^2 \left(1 - \frac{T}{2\pi M_P^2}\right)^2 d\theta^2 \quad (4.20)$$

which as expected is the metric of a conical spacetime with deficit angle T/M_P^2 .

5 Physical properties

In this section we wish to describe some of the physical properties of the ϕ and s strings presented in the previous section. These are rather different from the ordinary abelian Higgs

model strings.

The large r dependence of our solutions has a simple and important physical origin. The contribution to the energy from the angular derivatives is given by ($M_P = 1$),

$$\int r dr d\theta \frac{1}{r^2} \left[|\phi|^2 (\partial_\theta \Theta - q A_\theta)^2 + \frac{1}{s^2} (\partial_\theta a + 2\delta A_\theta)^2 \right] \quad (5.1)$$

where Θ is the phase of the tachyon. With our choice of normalization the two phases of the problem Θ and a have periodicities equal to 2π and $4\pi\delta$ respectively. It is then clear that for generic windings of m and n it is possible to cancel both gradient terms at infinity only if we allow the factors $|\phi|^2$ or $1/s^2$ to go to zero. In fact this is the behavior of the fields required by the BPS equations but we would like to stress that the argument only relies on the finite energy condition.

We therefore have three different asymptotic strategies to find finite energy solutions. We can cancel the gradient energy associated with the tachyon field by choosing the gauge field appropriately and consequently have $s \rightarrow \infty$ as $r \rightarrow \infty$. These are the ϕ -strings. If the axion does not wind ($m = 0$) the dilaton goes to a constant at the core implying, contrary to the standard abelian Higgs model, that the gauge symmetry is not restored there. The BPS equations however force the fields to be away from the vacuum at the core. At infinity the gauge symmetry is Higgsed by the tachyon field instead.

On the other hand, we can choose the gauge field such that it cancels the gradient term for the axion forcing $\phi \rightarrow 0$ as $r \rightarrow \infty$. These are the s -strings. These solutions resemble in some ways the stringy cosmic strings studied in the early nineties [26] which are also relevant for the description of the D_7 -brane [27]. In that case the only field is the complex dilaton $S = s + ia$ and finite energy supersymmetric configurations are achieved by using the $SL(2, \mathbb{Z})$ invariance of the action. The solution for S is a holomorphic function of the transverse coordinate $z = x + iy$. Close to the core,

$$S \approx -\log z \quad (5.2)$$

which is the same asymptotic solution as for the s -strings. The divergence of the dilaton at the core was interpreted in [26] as decompactification of the space or weak coupling. As we will see in the next section a similar understanding will be very useful to interpret our solutions as the D -strings of string theory.

The last possibility to achieve finite energy is to tune the winding numbers of the two fields in such a way that both gradients are canceled and therefore the string solutions can approach any point of the vacuum manifold at infinity.

The reader might have also noticed close similarities between the strings of this paper and semilocal strings (see [28] and Refs. therein). In the simplest semilocal strings there is a

complex Higgs doublet with equal $U(1)$ charges. String solutions can be found for example by setting one of the components of the doublet to zero. In the Bogomol'nyi limit the strings can also be made supersymmetric. In this case the stability of the strings is not guaranteed by the topology of the vacuum manifold which is S_3 since $\pi_1(S_3) = I$. The same holds in our solutions. Even though the vacuum manifold M in our case appears different at first sight, we still have $\pi_1(M) = I$. The axion phase becomes not well defined when $s \rightarrow \infty$ so the winding of the axion can be undone by taking $s \rightarrow \infty$ without leaving the vacuum manifold. These solutions are stable because of supersymmetry not because of topology.

Another typical feature of the semilocal strings in the BPS limit, is the presence of zero modes [29] beside those due to the breaking of spacetime symmetries. This is also present in our case since our solutions depend on an arbitrary integration constant k which controls the size of the string core. It is interesting to notice however that this does not lead to extra localized zero modes at least for the ϕ -strings. To compute the effective action of the string we promote the integration constant k to a field depending on the coordinates of the string. Considering the asymptotic expansion of the dilaton for the ϕ -strings, $s \sim \log r + k$ it is easy to see that the kinetic term for k is badly divergent (power-like). This implies that this mode is frozen, it cannot be excited with any finite energy excitation. It would be interesting to test numerically whether this mode can be excited in a collision of two strings or whether due to the power divergence of the kinetic term it is always frozen. For the s -strings the situation is different since f approaches the vacuum as a power law and therefore seems to be possible (and model dependent) to have an integrable bosonic zero mode in this case.

Finally, we should also mention the presence of localized fermionic zero modes. In the globally supersymmetric scenario there are at least two fermionic zero modes localized on the string due to partial supersymmetry breaking. These modes are the supersymmetric partners of the translation modes living on the string. Their profile can be obtained as usual by acting on the string background with the broken supersymmetry generators. In supergravity the story is rather different because, according to a well known argument due to Witten [30], unbroken supersymmetry does not guarantee fermi-bose degeneracy. In fact it can be seen [24] that the fermionic zero modes become non-normalizable when gravity is dynamical so they are projected out of the spectrum of the worldsheet theory.

6 D -brane interpretation

In this section we argue that the string solutions studied in this paper have a natural interpretation in terms of string theory solitons. To begin with let us review the key argument behind the conjecture of [5]. According to this conjecture, in four dimensions any non-singular BPS

solitonic representation of D -strings (admitting an asymptotically locally flat space) must be some sort of a D -term string. This can already be argued in a globally supersymmetric limit, but supergravity gives the following general argument (see section 4.4). Any finite tension (more precisely $T < M_P^2$) codimension two object, *irrespective* of its core structure, creates a conical angular deficit at infinity. Covariantly constant spinors cannot be defined on a conical spacetime, so generically all the supersymmetries are broken. The only way to avoid this conclusion is if the gravitino is charged under the $U(1)$ responsible for the string magnetic flux, because in this case the gravitino variation contains a piece proportional to gauge connection. The $U(1)$ in question is therefore a gauged R -symmetry. In $4D$ supergravity the necessary and sufficient condition for the gravitino to carry a $U(1)$ charge is the presence of a constant FI term ξ . This gives the gravitino a charge

$$q_\psi = \frac{\xi}{2M_P^2}. \quad (6.1)$$

Thus, if BPS D -strings admit a $4D$ solitonic description, they should be seen as D -term strings with a FI term. This is the key point of [5].

What is the physical origin of this FI term? We have seen, that in theories with GS anomaly cancellation there always is a dilaton-dependent contribution to the FI term. One may naively hope to convert this into a constant FI by stabilizing s . However, there is an obvious reason why such a program cannot work [8]. Even if one imagines that there can be a susy-preserving dynamics that stabilizes s , the same dynamics should also get rid of FI term at low energies. This is because s becomes part of a massive vector supermultiplet, and if we supersymmetrically integrate out s , consistency demands that the whole vector field is integrated out leaving no D -term at low energies. Thus, the constant FI term responsible for the BPS D -strings must be there "from the beginning". The natural candidate is the tension of the $D - \bar{D}$ system. This conjecture passes several consistency checks, in particular it reproduces the correct relation between the D -brane tensions. However, the solution presented in [5] was incomplete as BPS D -strings should also be charged under Ramond-Ramond fields which were not included in that solution. Because of this, the above conjecture left some puzzling questions, which our solutions clarify.

ϕ -String: Wrapped D_{1+q} -Branes

In ten dimensions D_p -branes are supersymmetric objects carrying a quantized charge of the C_{p+1} Ramond-Ramond gauge field. Upon compactification to four dimensions a D_{1+q} -brane can generate strings by wrapping a non trivial q -cycle of the internal manifold. These strings are will be charged under the two-form arising as the zero mode of the RR field to which the

D_{1+q} -brane couples⁶. In four dimensions a two-form is dual to a scalar so the D -string should be an axionic string. This naively leads to a puzzle because if the string carries an axionic charge δ , the energy density of the long range field should be

$$\frac{\delta}{r^2}, \quad (6.2)$$

which produces a logarithmic divergent energy per unit length. While the formally infinite energy of a single global string can be acceptable phenomenologically when several strings are included (such that the total axion charge is equal to zero) the infinite contribution to the energy of the axion would certainly destroy the BPS nature of the solutions. This would seem to contradict to the expectation that D -strings are BPS saturated objects.

The string solutions found in this paper have precisely the correct properties to solve this puzzle. To be concrete let us consider D_1 -strings produced in $D_3 - \bar{D}_3$ annihilation. When the two branes are far apart, that is at a distance greater than the string length l_s , this system breaks all the supersymmetries. In the worldvolume description supersymmetry is broken by the positive energy density of the branes. According to our description, this breaking is reproduced in the low energy effective action by a constant FI term. The normalization is given by (in string frame)

$$2T_3 = \frac{2}{g_S(2\pi)^3\alpha'^2} = \frac{g^2}{2}\xi^2, \quad (6.3)$$

where g_S is the string coupling and $\alpha' = l_s^2$ (we follow the conventions of [33]). When the branes come closer than l_s a complex tachyon develops in the spectrum of the open strings stretching between the brane-anti-brane pair, signaling an instability of the system. This is the origin of the tachyon ϕ in our effective action. As explained earlier the gauge symmetry of the $D_3 - \bar{D}_3$ pair is $U(1) \otimes U(1)$ and the tachyon is charged with respect to the diagonal subgroup. The annihilation proceeds through tachyon condensation in which the tachyon compensates the positive energy density restoring the supersymmetric vacuum without D_3 -branes. This is the essence of Sen's tachyon condensation [11]. In the annihilation process codimension two objects are produced which are D -strings. In our picture the strings produced in the annihilation are simply the ϕ -strings created by the winding of the phase of the tachyon. One can see that the tension has the same scaling as in string theory. In our solution,

$$T_1 = \frac{1}{g_S(2\pi)\alpha'} = \frac{2\pi\xi}{q}. \quad (6.4)$$

⁶In realistic compactifications, whether the D -strings will remain charged under the four dimensional RR two-form is model dependent as some zero modes may be projected out by the compactification [31]. Here we imagine a compactification where the relevant zero modes are not projected out.

Comparing with eq. (6.3) this implies the relation $q^2 g^2 = 8\pi g_S$. This formula has the correct scaling between the Yang-Mills and string coupling. The numerical coefficient is also reproduced for $q = 2$.

Let us now discuss the RR charges. The gauge field on the worldvolume of a D_3 -brane couples to the RR two-form C_2 via the Wess-Zumino coupling,

$$2\pi\alpha'g_S T_3 \int_{3+1} F_2 \wedge C_2. \quad (6.5)$$

This coupling means that a magnetic flux tube carries the RR charge of C_2 in ten dimensions, or in other words it is a D_1 -brane. In the $D_3 - \bar{D}_3$ system branes have opposite RR charges so that C_2 couples precisely to the same linear combination under which the tachyon is charged. By compactifying to four dimensions only the zero mode of C_2 is kept, leading to an effective action of the form,

$$\int d^4x [M_P^2 (dC_2)^2 + \xi F_2 \wedge C_2] \quad (6.6)$$

up to numerical factors and coupling to the dilaton. In order to connect our solutions with the string theory picture it is useful to dualize the axion a to a two-form C_2 in the lagrangian (2.2). The duality transformation is given by,

$$s^2 dC_2 = \star(da + 2\delta A) \quad (6.7)$$

through which the action takes the form (6.6). One can deduce that the axionic charge under the $U(1)$ scales as $\delta \approx \xi/M_P^2$ which as expected vanishes in the large volume limit.

The four dimensional RR charge of the string solutions is given by,

$$\int \star dC_2 \quad (6.8)$$

where the integral is taken over a circle surrounding the string. Using eq. (6.7) one can see that in all our solutions this integral is zero at infinity signifying that the RR charge, as measured with respect to the zero mode of C_2 , is always completely screened at large distances. Notice however that the integral is not zero at finite distances signaling the presence of an effective RR charge. This behavior, beside being forced by finite energy, has a natural interpretation in the full string theory. After compactification the four dimensional field s is a combination of the radius modulus and the ten dimensional dilaton. s going to infinity therefore corresponds to decompactification of the space (or alternatively the string coupling going to zero). This is similar to the effect advocated in the stringy cosmic strings [26] which also plays a crucial rôle in the F-theory [32]. Since the space is opening up new dimensions at large distances it is not surprising that the RR charge goes to zero as measured with the zero mode of the two-form C_2 .

Similar arguments can be used for the $D_{3+q} - \bar{D}_{3+q}$ system where q dimensions are wrapped on the internal manifold. In this case D_{1+q} objects will be produced, which from a four dimensional point of view are just D -strings. D_{1+q} branes couple to a C_{1+q} form. Upon dimensional reduction the D -strings will couple to a two-form arising as a zero mode of C_{1+q} so the analysis above can be repeated almost verbatim in this case.

The s -Strings: $D - \bar{D}$ Bound State?

Let us now turn to the s -strings. The above discussion set the stage for a simple interpretation of our s -string solution in the form of a BPS $D - \bar{D}$ bound state. In the s -strings the $D - \bar{D}$ energy is not compensated by the tachyon, but by the dilaton field. Since the tachyon is not condensed in the vacuum, this suggests that the branes do not annihilate but rather form some sort of boundstate. In ten dimensions the $D - \bar{D}$ system is unstable but this is not inconsistent with our findings since compactification is crucial in this case. In fact even in ten dimension a $D_2 - \bar{D}_2$ pair can be made supersymmetric by switching on a constant electric field on the worldvolume [19]. In our case it is the magnetic flux that stabilizes the configuration so it would be interesting to investigate the relation between the two solutions. We should also mention that since asymptotically s goes to a constant of order one, it might be that the effective theory breaks down due to strong coupling.⁷

Even though we have focused on the $D_3 - \bar{D}_3$ system we believe that our s -type string solutions are more generic and will exist in any compactification in which there is at least one surviving D_{3+q} -brane with the Wess-Zumino type coupling

$$\int_{3+q+1} F_2 \wedge C_{2+q}, \quad (6.9)$$

between the worldvolume $U(1)$ field and the bulk RR $2 + q$ -form. The topological reason for the existence of such strings is that in the effective $4D$ theory the $U(1)$ symmetry is Higgsed, provided the zero mode of C_{2+q} form is not projected out. This topological argument was already noticed in [34] and [7] but no smooth solutions were found. From above it follows that the tachyon plays an irrelevant role for the existence of the s -strings and its VEV can consistently be set to zero.

7 Conclusions and Outlook

The purpose of this paper was twofold. First, we tried to find explicit BPS-saturated string solutions in the supersymmetric abelian Higgs model coupled to an axion-dilaton multiplet.

⁷Note however that, since normally the existence of BPS objects visible at weak coupling can be extrapolated to strong coupling, it seems plausible that these solutions have an interpretation in general.

These are new BPS string solutions which arise in string motivated four dimensional models which have interest of their own. Secondly, we gave evidence that the strings in question have the basic features to describe the D -strings arising from unstable $D - \bar{D}$ systems in type II theories. In making this connection our guidelines were: 1) Sen's picture in which a D_{1+q} brane can be thought of as a vortex of the complex open string tachyon of a $D_{3+q} - \bar{D}_{3+q}$ pair; 2) The conjecture [5] that in 4D the BPS D -strings are the D -term strings formed by Higgsing the $U(1)$ gauge symmetry due to the presence of a constant FI term.

One important result, supporting our string theory interpretation, is that finite energy BPS solutions can only exist in the presence of a constant FI term in the effective action. Any field dependent FI term in isolation, for example those generated in heterotic string theory compactifications, cannot support smooth BPS strings because these configuration would have zero energy. The difference between the two cases arises from the fact that a field dependent FI term is generated from the gauging of the shift symmetry of the axion while the constant FI term only affects the potential (in global supersymmetry). We have also shown that the finiteness of energy forces the bosonic partner of the axion to vary along the radial direction. This behavior has a simple physical explanation. An infinite straight axionic string in four dimensions is analogous to an electric charge in 2+1 dimensions so it would naively have a logarithmically divergent energy per unit length. The only way to avoid this conclusion is if the charge is screened at large distances. This precisely achieved by the variation of the dilaton. The charge as measured from the flux goes to zero at large distances from the string so that the charge becomes completely screened.

We have found two qualitatively different classes of BPS string solutions which we have dubbed the ϕ -strings and the s -strings depending on whether the solution is supported by the tachyon field ϕ or the dilaton s . For the ϕ -strings the magnetic flux tube of the string is induced by the winding of the phase of the tachyon while for the s -strings what is relevant is the axion winding. For special values of the windings the two solutions merge into each other.

The coupling to supergravity was also discussed generalizing previous studies of [5, 24]. Under very general assumptions we have shown that BPS string solutions in flat space (with any Kähler potential for the chiral fields) remain supersymmetric even after the coupling to supergravity. As usual a finite energy configuration with codimension two produces an asymptotically conical space with deficit angle proportional to the tension of the object. The possibility to define Killing spinors in this space relies on a remarkable cancellation between the deficit angle and the Aharonov-Bohm phase as first noticed in 2+1 dimensions in [24]. We have shown that this effect is generic.

The main motivation for this work was the study of the connection between D -term strings and stringy D -strings along the lines of [5]. D -strings carry RR charges so it was our goal

to clarify how to couple the D -term strings to the RR fields. A string couples electrically to a two-form which is dual to an axion in four dimensions. In this paper we have shown that it is indeed possible to couple the abelian Higgs model strings to an axionic multiplet preserving their BPS nature.

The relation between D -strings and gauge theory strings has interesting consequences for string cosmology and phenomenology such as the fact that brane-anti-brane energy is represented by an FI D -term of the worldvolume theory. This observation relates D -term inflation [35] to D -brane inflation [3]. We hope to return to this and related issues in a future publication. Also having explicit solutions could be useful for a better understanding of the reconnection probability of cosmic D -strings and differentiating between ordinary gauge theory cosmic strings and the stringy ones [36].

In this paper we heavily relied on supersymmetry and BPS properties for two main reasons. On one hand we wished to obtain exact solutions at least in some approximation. On the other hand, supersymmetry was an useful tool for "tagging" the solutions in the $4D$ theory and tracing their origin in full string theory picture. In the real world, of course, supersymmetry is broken, and this breaking is expected to affect some properties of our solutions. For example, dilaton stabilization may create a divergent tension. In this case the strings will acquire some properties of global axionic cosmic strings, with an unusually small coupling to the axion. Due to this smallness, the dynamics of such strings will still be dominated by their core structure.

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